On fractal structure of quantum gravity and relic radiation anisotropy

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Abstract

It is argued that large-scale ($> 7^o$) cosmic microwave background anisotropy detected in COBE cosmic experiment can be considered as a trace of the quantum gravity fractal structure.

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1 Introduction

It seems that a fractal structure is an intimate property of the Universe. The superclusters (large clusters of galaxies containing up to hundreds of thousands of galaxies) of size about 50 Mpc are separated by almost void space: the mean distance between two superclusters is about 100 Mpc. Clusters of galaxies (the typical cluster size is about 5 Mpc) containing hundreds of galaxies are, in their turn, separated by voids about few Mpc. This fractal hierarchy can be easily traced up to subnuclear scales (10^{-13}cm.). Quantitatively, the large-scale fractal structure of the Universe can be described in terms of the mass interior to the spherical volume of certain radius r. The typical dependence, measured by observing 21 cm hydrogen emmission of gas clouds moving around the galaxy is

$$\mathcal{M}(r) \propto r^{\alpha}, \qquad \alpha \approx 1, \tag{1}$$

whereas a luminous mass associated with the light would supply only $r^{-1/2}$. It is commonly accepted that an additional mass in the form of non-luminous dark matter [1]. Since $\alpha < 3$ in the power law (1), we have a typical mass distribution on a fractal set embedded in D=3 space.

On the other hand, one of the most important recent developments in gravity theory was related to the fractal based regularization of quantum gravity [3]. In view of this one may believe

that a fractal structure is a fundamental property of physical space-time itself.

In this note we interpret the COBE satellite data on the anisotropy of cosmic microwave background radiation (CMBR) as a possible manifestation of fractal structure of the Universe.

2 On the discrete symmetry in quantum gravity

A regularization of two-dimensional quantum gravity, made by V. G. Knizhnik, A. M. Polyakov and A. B. Zamolodchikov (KPZ), comes from the fact that continuum formulation [2] and the dynamical triangulation [5] are equivalent. On the basis of the Polyakov regularization procedure [7], where the position of the surface in the embedding space X_μ and the internal surface geometry (g_{ab}) are treated as independent fields, one can construct a Nambu-like action

$$
\begin{aligned}
&S[X_\mu, g_{ab}] = \\
&= \tfrac{1}{2}\textstyle\int_M g_{ab}\frac{\partial X_\mu}{\partial \xi_a}\frac{\partial X_\mu}{\partial \xi_b}\sqrt{\det g}\; d^2\xi + \beta \int_M \sqrt{\det g}\; d^2\xi + \qquad (2)\\
&+\text{fermion terms}
\end{aligned}
$$

where $\xi = (\xi_1, \xi_2)$ is the parametrization of the manifold M defined by function $X_\mu = X_\mu(\xi)$. This or a similar Nambu-Goto action usually stands in the string functional integral taken with respect to both independent fields X and g.

In the dynamical triangulation [5] of 2D quantum gravity, as well as in higher dimensional versions [6], the path integral over internal metric g_{ab} is replaced by summation of all the different types of surface configurations with given number of triangles. For the sake of preserving reparametrization invariance after dis-

cretization [5], the topology of the manifold M is usually specified as the sphere S^2 [5, 10]. The partition function takes the form

$$Z(A) = \int_M \mathcal{D}X\mathcal{D}g \exp(-S), \tag{3}$$

or its discrete counterpart [10]

$$Z_{reg}(A) = \sum_G Z_m(G)\delta_{Na^2,A}, \tag{4}$$

where A is the total area, N is the number of equilateral triangles and a^2 is the area of an triangle. The matter part of the partition function $Z_m(G)$ comes from the fermion term of the KPZ lagrangian

$$\mathcal{L} = \bar{\phi} v^{\alpha a}\gamma^a \partial_\alpha \phi, \tag{5}$$

where $v_{\alpha a}$ are ordinary "zweibeins".

Formally substituting functional integral (3) by its discrete counterpart (4) we need to sum over all possible triangulations of S^2. Practically, we are to impose some additional conditions to avoid summation over singular triangulations, i. e. triangulations which include links with coinciding ends. Referring the reader to [5, 11] for detailed study of triangulations and fractal properties of related partition functions, we shall concentrate on some of their properties significant for the phenomenological applications.

First. The triangulation procedure can be extended to an S^n sphere [6], which, as a boundary of $(n+1)$-dimensional simplex, can be divided into n-dimensional simplices.

Second. From the conformal invariance standpoint, of all the subdivisions of S^n the subdivision into equilateral simplices is preferable.

Third. The whole partition function (3) is related to the physical object, which is isotropic (in the sense of having no preferable direction on S^n), but may have a discrete symmetry group, and hence, have certain distinguished correlation angles. For example, if we sum over all possible triangulations of S^2

using equilateral triangles, the correlations of any observables depending on matter fields increase at angles $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ because of Z_3 symmetry group. Similarly, the correlations should increase at tetrahedron group angles when S^3 is considered.

Fourth. The two-dimensional quantum gravity can be regarded as only the simplest case of extended object physics. However, when reducing the physics from arbitrary n-dimensional space to $(n-1)$ dimensions we restrict S^n triangulation with n-dimensional simplices to S^{n-1} triangulation with $(n-1)$ - dimensional ones, because an $(n-1)$-dimensional simplex is a boundary of an n-dimensional one. Thus, for the case of equilateral symplices we should always have Z_3-symmetry in $D = 2$, or tetrahedron symmetry in $D = 3$.

3 Discrete symmetry as a possible source of relic radiation anisotropy

Let us consider the data [12] on relic radiation anisotropy. The relic microwave radiation ($T = 2.73^o K$) was not significantly affected by the late-stage processes in the Universe, that is why its amplitudes depend mostly on the early Universe parameters. It is worth to note, that the large scale anisotropy of relic radiation, found in COBE and RELICT-1 experiments, has a rather small value $\frac{\Delta T}{T} \sim 10^{-5}$, but a high confidence level — up to 90%, including systematic errors [12, 15].

Of course, the first aim of the observers in both COBE and RELICT experiments was to measure the dipole and quadrupole components of microwave background [12] and to test the existence of anomalous signal over the mean background [15]. Basing on the COBE experiment data, the autocorrelation function

$$C(\alpha) = \langle \Delta T(\theta) \Delta T(\theta + \alpha) \rangle \tag{6}$$

has been obtained. Here α is the angle separation and θ is an angular coordinate on certain two-dimensional plane. Fig. 1

Qualitatively, the behavior of the relic signal autocorrelation function (See fig.1) is the following: it has a sharp maximum , it has another maximum localized at α close to 120 degrees, and it has two minimums at 60^o and 180^o. (Maximum at *alpha* close to 90^o is possibly related to quadrupole component of CMBR and is less confidient [13]). The behavior of the autocorrelation functions is just atmost the same for the data obtained at frequencies 53 GHz and 90 GHz [12].

Correlation function (6) has been studied in [14] in connection with present cosmological models. In particular, an attempt has been made to compare the COBE data with certain Dark Matter (DM) models. This comparison does not suit well. For instance, the relic density anisotropy given by Holtzman model [16] increases monotonously with α increasing from 60 to 180 degrees [14].

Taking into account all the mentioned arguments, we interpret the regularities of autocorrelation function (6) behavior, as a manifestation of Z_3-symmetry. The presense of Z_3-symmetry does not imply n preferable directions in space here, instead we have a preferable separation angle. It should be mentioned that in COBE theoretical study [14] the best line fit for autocorrelation function (6) was taken in the form

$$C(\alpha) = A + B\cos\alpha + C_M^0 \exp\left[-\frac{\alpha^2}{2\sigma^2}\right], \tag{7}$$

though the locations of autocorrelation function maximums at 0^o and 120^o and minimums at 60^o and 180^o suggest more direct parametrization

$$C(\alpha) = A + B\cos 3\alpha + C_M^0 \exp\left[-\frac{\alpha^2}{2\sigma^2}\right] \tag{8}$$

4 On the flat-space limit of simplicial quantum gravity

The simplest way to imagine how the distribution of relic radiation with the simplicial symmetry could emerge from space-time geometry is that of simplicial quantum gravity [19, 20]. This theory enables one to describe only pure gravity without matter fields in a consistent way. Exact solution for matter coupling has been found only for two-dimensional case [4, 5]. For higher dimensions, if we want to describe the Nature as it is, we are to face a lot of matter coupling problems. The question of our particular concern should be the existence of flat-space continuous limit. Hereinafter, we analyse the problems arising in $D > 2$ simplicial quantum gravity continuous limit and suggest a way to avoid them.

The very fact which, in our opinion, lead to KPZ regularization of two-dimensional quantum gravity was the fractal nature of dynamically triangulated surface, rather than simplicial structure itself. (Investigation, in some way similar to it, has been performed by Crane and Smolin [21] without using triangulation at all.) That is why we should expect some fractal structure which enables one to remove the divergences.

Indeed, in direct studies of quantum field theory models on fractal space-time [22], as well as in studies devoted to fractal lattices [23], it has been shown that fractal sets, being scale-invariant, are essentially relevant for treating by the renormalization group (RG) technique. Unfortunately, the price for well defined scale properties is the lack of translation invariance. This leads to the divergences. Till now this obstacle has not been completely overcome.

As we consider the flat-space limit of (Euclidean) simplicial gravity, we must pay attention to those fractal sets, which are suitable for triangulation of an S^n sphere. Thus, we consider *Sierpinski hypergasket*, a generalization of the Sierpinski gasket

constructed in two dimensions. Let us recall the construction procedure [22]. Fig. 2

Partitioning the unit d-simplex in R^d into (d+2) sub-simplices of edge-length 1/2 one (i) removes the open central sub-simplex; (ii) repeates the operation with the $(d+1)$ closed sub-simplices.

Sierpinski gaskets obtained in this way can be used for triangulation of an S^n-sphere. Their self-similarity is much relevant to RG applications. Their shortcomings are also evident. They are not invariant under translations, even inside a single gasket, and they are not dense in the embedding space. That is why we are looking for the simplicial fractal set better in this relation.

Let us modify the gasket generating procedure. To clarify the consideration, let us imagine a unit simplex of black color. On the first step of the recursive procedure we remove the central open part of it, the central sub-simplex becomes "white", and then — here is the difference — repeat the procedure with *all* $(d+2)$ sub-simplices. The generalization to "white" pieces seems evident: the central part of each simplex reverses its color. Fig. 3

Since the numbers of "black" and "white" sub-simplices at the $(k+1)$ stage of the recursive procedure are

$$\begin{aligned} n_W^{k+1} &= (d+1)n_W^k + n_B^k \\ n_B^{k+1} &= (d+1)n_B^k + n_W^k, \end{aligned} \tag{9}$$

for asymptotically large k we obtain

$$n_k \approx \frac{1}{2}(d+2)^k$$

simplices of each color of $\delta = 2^{-k}$ edge-size. The fractal dimension of the constructed set is

$$D = \frac{\log(d+2)}{\log 2}, \tag{10}$$

rather than

$$D = \frac{\log(d+1)}{\log 2}, \tag{11}$$

for Sierpinski gasket, But the scaling law is identical:

$$2^k \cdot \mathcal{G}(d) = \mathcal{G}(d). \tag{12}$$

As far as we know, there is no commonly accepted name to such a set. Here, as the relation between black and white in the construction is much like to ancient Chinese symbol of Yin and Yang, we can informally call it Yin-Yang-gasket.

The geometrical properties of the above constructed set as a building block for a piecewise approximation of an S^n sphere ($n > 2$) require further investigation. Nevertheless, we can already mention the properties which could revive the simplicial quantum gravity phenomenology. The gasket is *(i) simplicial, (ii) scale-invariant, (iii) homogeneous, (iv) dense in embedding space.*

5 Conclusion

The data on relic radiation anisotropy obtained by both RELICT and COBE groups are worth further deep investigation. Nonetheless, even the results already obtained from data processing seem to be in good agreement with the hypothesis of discrete symmetry of space-time arising in fractal quantum gravity. Other cosmological data, e.g. mass distribution, also do not contradict either possible fractal structure of the Universe. It might be argued, that both the tetrahedron symmetry, if found, and the fractal structure of the visible Universe, can be regarded as an argument for the existence of cosmic strings [17]. Indeed, cosmic strings, as topological defects which could be formed at a phase transition in the early Universe, can have a number of cosmological applications. In particular, they can form a network with a fractal structure having tetrahedron symmetry [18]. Though the question, why cosmic strings must form the tetrahedron structure still remains.

Therefore, some new tests for possible discrete symmetry can be proposed. The simplest among them are: (*i*) to test $\cos n\alpha, \quad n > 1$ in (7) for other Z_n groups, (*ii*) to use COBE and RELICT data to search for the tetrahedron or other essentially three-dimensional space symmetry groups.

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References

[1] R.Sancisi and T.S. van Albada, in: *Dark Matter in the Universe*, eds J.Kormendy and G.Knapp (Reidel, Dordrecht, 1987) p.67

[2] A.M.Polyakov, *Mod.Phys.Lett* A 2(1987)893.

[3] V.G.Knizhnik , A.M.Polyakov and A.B.Zamolodchikov, Modern. Phys. Lett. A 3(1988)819.

[4] V.A.Kazakov, I.K. Kostov and A.A.Migdal, Phys. Lett. B 157(1985)295.

[5] D.V.Boulatov, V.A.Kazakov, I.K.Kostov and A.A.Migdal, Nucl. Phys. B 275(1986)641.

[6] J.Ambjørn, J. Jurkiewicz and C.F. Kristjansen, Nucl.Phys. B393 (1993) 601.

[7] A.M.Polyakov, Phys. Lett. B 103(1981)207.

[8] E.Brezin and V.A.Kazakov, Phys. Lett. B 236(1990)114.
M.Douglas and S.Shenker, Nucl. Phys. B335(1990)635.
D.J.Gross and A.A.Migdal, Phys. Rev. Lett. 64(1990)127.

[9] M.E.Agishtein and A.A.Migdal, Int. J. Mod. Phys. C 1(1990)167; Nucl. Phys. B350(1991)690.

[10] N.Kawamoto, *Institute for Nuclear Study of Tokyo*, Preprint INS-Rep. 972 (1993)

[11] N.Kawamoto, V.A.Kazakov, Y.Saeki and Y.Watabiki, Phys. Rev. Lett. 68(1992)2113.

[12] G.Smoot *et al.,* Astrophys. J. 396(1992)L1.

[13] C.L.Bennet *et al.,* Astrophys. J. 396(1992)L7.

[14] E.L.Wright *et al.,* Astrophys. J. 396(1992)L13.

[15] I.A.Strukov, A.A. Brukhanov, D.P. Skulachev and M.V. Sazhin, Phys.Lett. B 315(1993)198.

[16] J.A.Holtzman, Astrophys. J. Suppl. 71(1989)1

[17] T.W.B.Kibble, J.Phys. A 9(1976)1387.

[18] M.Aryal, A.E. Everett and A.Vilenkin, Phys. Rev. D 34(1986)434.

[19] J.Ambjørn and J. Jurkiewicz, Phys.Lett B 278 (1992) 42.

[20] M.E. Agishtein and A.A. Migdal, Mod. Phys. Lett. A7 (1992) 1039.

[21] L. Crane and L.Smolin, Nucl. Phys. B 267(1986)714.

[22] Eyink G., Comm. Math. Phys. 125(1989)613.
Eyink G., Comm. Math. Phys. 126(1989)85.

[23] Y. Gefen, B. Mandelbrot and A. Aharony, Phys. Rev. Lett. 45(1983)855; Phys. Rev. Lett. 50(1983)145

Figure captions

Fig. 1 Correlation functions $C(\alpha)$, at various Gallactic lattitude cuts for the 53MHz map. (Reprinted from [12]).

Fig. 2 Second stage of the $(2D)$ Sierpinski gasket construction.

Fig. 3 Second stage of the $(2D)$ *Yin-Yang-gasket* construction. The difference from fig.2 is shown in grey.